Appendix B

Formules utiles, analyse vectorielle

B.1 Coordonnées cartésiennes

(B.1)
$$\overrightarrow{\text{grad}}\ U = \frac{\partial U}{\partial x}\vec{e}_x + \frac{\partial U}{\partial y}\vec{e}_y + \frac{\partial U}{\partial z}\vec{e}_z$$

(B.2)
$$\text{div}\,\vec{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$$

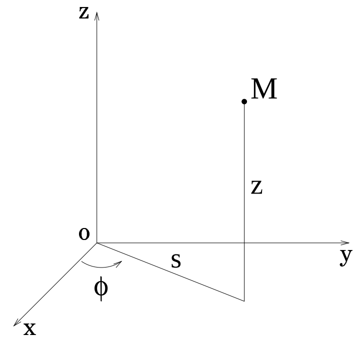
(B.3)
$$\overrightarrow{\text{rot}}\ \vec{V} = \left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right]\vec{e}_x + \left[\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right]\vec{e}_y + \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]\vec{e}_z$$

(B.4)
$$\Delta U = \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2}$$

(B.5)
$$\Delta\vec{V} = [\Delta V_x]\ \vec{e}_x + [\Delta V_y]\ \vec{e}_y + [\Delta V_z]\ \vec{e}_z$$

(B.6)
$$\underline{\underline{\nabla}}\vec{V} = \begin{bmatrix} \frac{\partial V_x}{\partial x} & \frac{\partial V_x}{\partial y} & \frac{\partial V_x}{\partial z} \\ \frac{\partial V_y}{\partial x} & \frac{\partial V_y}{\partial y} & \frac{\partial V_y}{\partial z} \\ \frac{\partial V_z}{\partial x} & \frac{\partial V_z}{\partial y} & \frac{\partial V_z}{\partial z} \end{bmatrix}$$

B.2 Coordonnées cylindriques



(B.7)
$$\overrightarrow{\text{grad}}\ U = \frac{\partial U}{\partial s}\vec{e}_s + \frac{1}{s}\frac{\partial U}{\partial \phi}\vec{e}_\phi + \frac{\partial U}{\partial z}\vec{e}_z$$

(B.8)
$$\text{div}\,\vec{V} = \frac{1}{s}\frac{\partial}{\partial s}(sV_s) + \frac{1}{s}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$$

(B.9)
$$\overrightarrow{\text{rot}}\ \vec{V} = \left[\frac{1}{s}\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right]\vec{e}_s$$

$$+ \left[\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right]\vec{e}_\phi$$

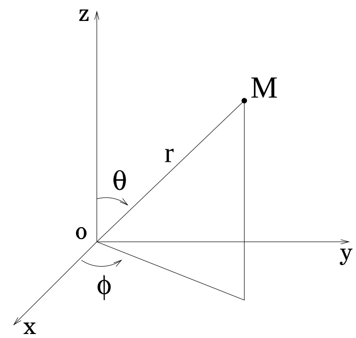
$$+ \left[\frac{1}{s}\frac{\partial}{\partial s}(s\,V_\phi) - \frac{1}{s}\frac{\partial V_s}{\partial \phi}\right]\vec{e}_z$$

(B.10)
$$\Delta U = \frac{\partial^2 U}{\partial s^2} + \frac{1}{s}\frac{\partial U}{\partial s} + \frac{1}{r^2}\frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2}$$

(B.11)
$$\Delta\vec{V} = \overrightarrow{\text{grad}}\ \text{div}\vec{V} - \overrightarrow{\text{rot}}\,\overrightarrow{\text{rot}}\ \vec{V}$$

(B.12)
$$\underline{\underline{\nabla}}\vec{V} = \begin{bmatrix} \frac{\partial V_s}{\partial s} & \frac{1}{s}\frac{\partial V_s}{\partial \phi} - \frac{V_\phi}{s} & \frac{\partial V_s}{\partial z} \\ \frac{\partial V_\phi}{\partial s} & \frac{1}{s}\frac{\partial V_\phi}{\partial \phi} + \frac{V_s}{s} & \frac{\partial V_\phi}{\partial z} \\ \frac{\partial V_z}{\partial s} & \frac{1}{s}\frac{\partial V_z}{\partial \phi} & \frac{\partial V_z}{\partial z} \end{bmatrix}$$

B.3 Coordonnées sphériques



(B.13) $$\overrightarrow{\text{grad}}\ U = \frac{\partial U}{\partial r}\vec{e}_r + \frac{1}{r}\frac{\partial U}{\partial \theta}\vec{e}_\theta + \frac{1}{r\ \sin\theta}\frac{\partial U}{\partial \phi}\vec{e}_\phi$$

(B.14) $$\text{div}\,\vec{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,V_r\right) + \frac{1}{r\ \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\,V_\theta\right) + \frac{1}{r\ \sin\theta}\frac{\partial V_\phi}{\partial \phi}$$

(B.15) $$\overrightarrow{\text{rot}}\ \vec{V} = \frac{1}{r\ \sin\theta}\left[\frac{\partial}{\partial\theta}\left(\sin\theta\,V_\phi\right) - \frac{\partial V_\theta}{\partial \phi}\right]\vec{e}_r$$
$$+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r\ V_\phi\right)\right]\vec{e}_\theta$$
$$+\frac{1}{r}\left[\frac{\partial}{\partial r}\left(r\ V_\theta\right) - \frac{\partial V_r}{\partial \theta}\right]\vec{e}_\phi$$

(B.16) $$\Delta U = \frac{\partial^2 U}{\partial^2 r} + \frac{2}{r}\frac{\partial U}{\partial r} + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial U}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial^2\phi}$$

(B.17) $$L_2\,U = -\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial U}{\partial\theta}\right) - \frac{1}{\sin^2\theta}\frac{\partial^2 U}{\partial \phi^2}$$

(B.18) $$\Delta\vec{V} = \overrightarrow{\text{grad}}\ \text{div}\vec{V} - \overrightarrow{\text{rot}}\,\overrightarrow{\text{rot}}\ \vec{V}$$

(B.19) $$\underline{\nabla}\vec{V} = \begin{bmatrix} \dfrac{\partial V_r}{\partial r} & \dfrac{1}{r}\dfrac{\partial V_r}{\partial\theta} - \dfrac{V_\theta}{r} & \dfrac{1}{r\sin\theta}\dfrac{\partial V_r}{\partial\phi} - \dfrac{V_\phi}{r} \\ \dfrac{\partial V_\theta}{\partial r} & \dfrac{1}{r}\dfrac{\partial V_\theta}{\partial\theta} + \dfrac{V_r}{r} & \dfrac{1}{r\ \sin\theta}\left(\dfrac{\partial V_\theta}{\partial\phi} - V_\phi\cos\theta\right) \\ \dfrac{\partial V_\phi}{\partial r} & \dfrac{1}{r}\dfrac{\partial V_\phi}{\partial\theta} & \dfrac{1}{r\ \sin\theta}\left(\dfrac{\partial V_\phi}{\partial\phi} + V_r\ \sin\theta + V_\theta\ \cos\theta\right) \end{bmatrix}$$

B.4 Identités vectorielles

(B.20) $$\mathrm{div}(\overrightarrow{\mathrm{grad}}\, U) = \Delta U$$

(B.21) $$\overrightarrow{\mathrm{rot}}\, \overrightarrow{\mathrm{grad}}\, U = \vec{0}$$

(B.22) $$\mathrm{div}\ \overrightarrow{\mathrm{rot}}\ \vec{V} = 0$$

(B.23) $$\overrightarrow{\mathrm{grad}}\ (U_1 U_2) = U_1\ \overrightarrow{\mathrm{grad}}\ U_2 + U_2\ \overrightarrow{\mathrm{grad}}\ U_1$$

(B.24) $$\mathrm{div}(U\vec{V}) = U\ \mathrm{div}\vec{V} + (\overrightarrow{\mathrm{grad}}\ U) \cdot \vec{V}$$

(B.25) $$\mathrm{div}(\vec{V_1} \wedge \vec{V_2}) = \vec{V_2} \cdot (\overrightarrow{\mathrm{rot}}\ \vec{V_1}) - \vec{V_1} \cdot (\overrightarrow{\mathrm{rot}}\ \vec{V_2})$$

(B.26) $$\overrightarrow{\mathrm{rot}}\ (U\vec{V}) = U\ \overrightarrow{\mathrm{rot}}\ \vec{V} + (\overrightarrow{\mathrm{grad}}\ U) \wedge \vec{V}$$

(B.27) $$\overrightarrow{\mathrm{rot}}\ (\vec{V_1} \wedge \vec{V_2}) = \vec{V_1}\mathrm{div}\vec{V_2} - \vec{V_2}\mathrm{div}\vec{V_1} + (\vec{V_2}\cdot\ \overrightarrow{\mathrm{grad}})\vec{V_1} - (\vec{V_1}\cdot\ \overrightarrow{\mathrm{grad}})\vec{V_2}$$

(B.28) $$\overrightarrow{\mathrm{rot}}\ (\overrightarrow{\mathrm{rot}}\ \vec{V}) = \overrightarrow{\mathrm{grad}}\ (\mathrm{div}\vec{V}) - \Delta\vec{V}$$