Dissipation mechanisms for convection in rapidly rotating spheres and the formation of banded structures

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We report banded zonal structures in numerical simulations of weakly nonlinear rapidly rotating convection. A quasigeostrophic model of convection is used to demonstrate how, in the presence of Ekman pumping, banded structures can develop immediately above the onset of convection, and in the absence of developed turbulence. We argue that these bands necessarily correspond to a regime in which both Ekman pumping and bulk viscosity equally affect the zonal flow and that their width scales with the Ekman number E as $E^{1/4}$. © *2006 American Institute of Physics*.
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We investigate the respective role of two dissipation mechanisms, bulk viscosity and Ekman friction, acting on weakly nonlinear rapidly rotating convection. To make the problem tractable in a parameter regime of small Ekman numbers (low viscosity), we use a simplified approach relying on a z-integrated set of equations (known as quasigeostrophic convection, or β convection[1]). In a previous investigation using a similar approach,[2] we have shown, in the presence of bulk viscosity only, that as the Ekman number is decreased toward realistic values (which are presently out of reach of fully three-dimensional modeling) the zonal flow becomes increasingly important near the onset of convection. We have shown that, from an asymptotic point of view, the zonal flow is strong enough to suppress convective motions on a quasiperiodic basis and to yield relaxation oscillations immediately above the onset.[2]

In all studies of quasigeostrophic or fully three-dimensional convection, when only bulk viscosity is retained, the zonal flow near onset takes the form of a large "S" structure in the radius, with two counter-rotating bands (retrograde near the axis and prograde near the equator). As the Rayleigh number is further increased above critical, the number of bands slowly increases.[3]

Recently, Jones *et al.*[4] investigated the effect of dissipation in the viscous boundary layers. They used a two-dimensional rotating annulus model with mixed mechanical boundary conditions. Their model is strongly nonlinear and relies on a prescribed radial variation of the stream function. Introducing the effect of Ekman pumping, they were able to achieve a significantly higher number of bands (up to six on the β plan) for a given Rayleigh number. The role of Ekman pumping in producing these jets still needs to be clarified, as well as the resulting balance between dissipation mechanisms. This turbulent approach differs from the weakly nonlinear investigation presented here.

The zonal flow is driven by nonlinearities. Independent of which dissipation mechanism affects the axisymmetric component of the flow, its radial profile exhibits prograde and retrograde velocities. It is the shearing action of the zonal flow on the convective columns which yields the kinetic energy saturation. The zonal flow has to produce a strong enough shear to saturate the growth of convective motions. If only bulk viscosity is retained in the zonal flow equation, this shear will be achieved while minimizing the second derivative of the zonal velocity in the same time. Close to the onset, this yields a large-scale profile for the zonal flow. Retaining only Ekman pumping in the equation of the zonal flow, the shear is achieved while minimizing the amplitude of the zonal flow. To achieve a smaller amplitude while maintaining the shear, it is necessary to oscillate on a smaller scale. This yields a banded structure. The combined effect of both dissipations (bulk viscosity and Ekman pumping) is described later in the paper and provides a banded zonal flow as well, but of larger scale.

We consider motions driven by buoyancy in a rotating spherical shell with a uniform distribution of internal heat sources. The rotation rate is Ω, ν is the kinematic diffusivity of the fluid, κ is its thermal diffusivity, and α its coefficient of thermal expansion. The gravity field is assumed to be purely radial and corresponds to a self-gravitating fluid: $\boldsymbol{g}=-g\boldsymbol{r}$. We further introduce θ, the deviation from the basic temperature profile of pure conduction ($T_s=T_0-\tilde{\beta}r^2/2$), and $-\tilde{\beta}r$, the temperature gradient in the absence of convection. Setting the unit of time to r_o^2/ν, the unit of length $\mathcal{L}$ to the outer sphere radius r_o, and the unit of temperature to $\tilde{\beta}r_o^2\nu/\kappa$, we introduce the following dimensionless parameters:

$$E=\frac{\nu}{2\Omega r_o^2},\quad Ra=\frac{\alpha\tilde{\beta}gr_o^6}{\nu\kappa},\quad Pr=\frac{\nu}{\kappa},$$

namely the Ekman number, the Rayleigh number, and the Prandtl number, respectively. The Prandtl number is set to unity for all simulations reported here. The aspect ratio r_i/r_o is fixed to 0.2. This small inner core leaves enough room for bands to develop outside the tangent cylinder.

Dissipative mechanisms in a rapidly rotating flow constitute a subtle issue. From a formal point of view, viscous dissipation affects the flow in two ways. First comes the bulk effects of viscous dissipation. Because of the very low Ekman number values associated with rapid rotation, this term only becomes significant at small scale. Dimensional analysis reveals scales such as $\mathcal{O}(\mathcal{L}E^{1/3})$ for z-aligned structures, such as vertical shear layers or convection near the onset.[1,5,6] However, viscosity is also essential near boundaries, where very sharp $\mathcal{O}(\mathcal{L}E^{1/2})$ layers develop. These layers actively affect the mainstream flow through Ekman pumping. This yields an additional dissipative term on the mainstream equations, known as Ekman friction or Ekman pumping.[7] This term will equally affect all scales of the mainstream flow. Its amplitude is controlled only by the Ekman number and the flow velocity. As a result, Ekman pumping will provide the dominant dissipation mechanism on the large-scale mainstream flow, for which the effects of bulk viscosity are vanishingly small.

We will investigate convection very close to the onset, and adopt (as in Ref. 2) a z-integrated weakly nonlinear formalism. The resulting domain consists of the gap between two concentric circles on which the velocity vanishes. Due to the computing domain geometry (which is not simply connected), we consider separately the mean zonal flow u_0 and all the nonaxisymmetric modes (see Ref. 8 for a discussion of this issue). In the weakly nonlinear approach, only one of the nonaxisymmetric modes is retained and expressed in the form of a stream function $\psi=\psi(s)e^{im\varphi}$, $m\in\mathbb{R}$ (see Ref. 2 for more details). The mode m is set to its critical value at the onset of convection, computed for each particular value of the Ekman number. The vorticity of the flow is then given by $\omega=-\Delta\psi+\mathbf{e}_z\cdot\nabla\ \ (u_0\mathbf{e}_\varphi)$. The resulting set of equations is

$$Pr\left(\frac{\partial\theta}{\partial t}+(\boldsymbol{u}\cdot\nabla)\theta\right)=\Delta\theta+\frac{\partial\psi}{\partial\varphi}, \tag{1a}$$

$$E\left(\frac{\partial}{\partial t}\Delta\psi-(\boldsymbol{u}\cdot\nabla)\omega\right)=E\Delta^2\psi+\frac{1}{1-s^2}\frac{\partial\psi}{\partial\varphi}+Ra\,E\frac{\partial\theta}{\partial\varphi}, \tag{1b}$$

$$E\left(\frac{\partial u_0}{\partial t}+(\boldsymbol{u}\cdot\nabla)u_\varphi|_0\right)=\underbrace{E\left(\Delta-\frac{1}{s^2}\right)u_0}_{B-V}-\underbrace{\frac{E^{1/2}}{2(1-s^2)^{3/4}}u_0}_{E-P}. \tag{1c}$$

The linear theory for the onset of convection in a rotating sphere indicates that convection develops at the onset on a horizontal length scale $\mathcal{O}(E^{1/3})$, both in the radial (s) and azimuthal (φ) direction.[9] On such length scale, the bulk viscosity acting on the nonaxisymmetric velocity can be estimated to scale as $E\times E^{-2/3}=E^{1/3}$, whereas the Ekman pumping will scale as $E^{1/2}$. For this reason, an asymptotically valid approximation is to retain only bulk viscosity as dissipating term in (1b), here in the form of a bi-Laplacian on ψ. On such small length scales, the Ekman pumping term is asymptotically negligible compared to bulk viscosity. Indeed, Ekman pumping only provides an $\mathcal{O}(E^{1/6})$ correction term on the critical parameters for the onset.[10]

Two dissipation terms are retained instead for the axisymmetric flow described by Eq. (1c): the bulk viscosity (marked "*B*-*V*") and the Ekman pumping term (marked "*E*-*P*"). The axisymmetric flow is obviously large scale in the φ direction: $\mathcal{O}(\mathcal{L})$. Its scaling in the radial s direction however remains to be determined. None of these dissipating terms can therefore be ruled out on the basis of a simple scaling argument. Note that we follow here an approach introduced by Jones *et al.*,[4] although our model is simplified by coupling two modes only and dropping the effect of pumping on the small scales ψ. System (1) can be solved using explicit mode coupling in the spectral domain. This weakly nonlinear approach is severely truncated in φ by only retaining two azimuthal modes, but its simplicity allows a much wider variation in parameters than the full modeling.

We will present results of numerical simulations retaining either the *B*-*V* term, or the *E*-*P* term, or both, in order to assess the role of the various dissipating processes (bulk viscosity or Ekman pumping, respectively).

We first verified that the introduction of Ekman pumping did not affect the qualitative result reported in Ref. 2, i.e., relaxation oscillations occur increasingly close to the onset of convection as E is decreased. Indeed, most of the flows we will investigate here are time dependent. We will now focus our attention on the effect of Ekman pumping on the zonal flow structure. Let us first consider the zonal flow at fixed Ekman number (here $E=10^{-6}$) and with increasing Rayleigh numbers (from 1.1 to 5 times critical). The corresponding results (zonal flows as functions of the radius) are displayed in Fig. 1(a).

First, in the presence of bulk viscosity only (the *B*-*V* term), the expected behavior ("S"-shaped zonal flow and slow increase in the number of bands with the Rayleigh number) is reproduced with this simplified model (see the left column of Fig. 1(a)). When, on the other hand, Ekman pumping provides the only dissipating term on the zonal flow (corresponding to a large-scale assumption on this flow), a large number of bands is produced in the system (see the central column of Fig. 1(a), and 1(b)). The bands become more numerous as the Rayleigh number is increased, here reaching up to seven bands for . It is however important to ponder at this stage the radial scaling of the bands produced with this model. Equation (1c) now involves no regularizing term in radius. As a result, the radial scale of u_0 will be directly provided by the energy input term, i.e., by ψ. It follows that near the onset u_0 will then scale radially as ψ, i.e., $\mathcal{O}(E^{1/3}\mathcal{L})$. One should also notice that banded solutions obtained in the presence of the *E*-*P* term generally have lower peak velocities than the large structures obtained with the *B*-*V* term only (see also the rms values). This strengthens the proposed physical interpretation, by which the level of shear necessary to saturate convection is achieved through a smooth large-scale (and therefore large-amplitude) profile in the case of bulk viscosity only, and through a small-amplitude (and therefore small-scale) profile in the presence of Ekman pumping.

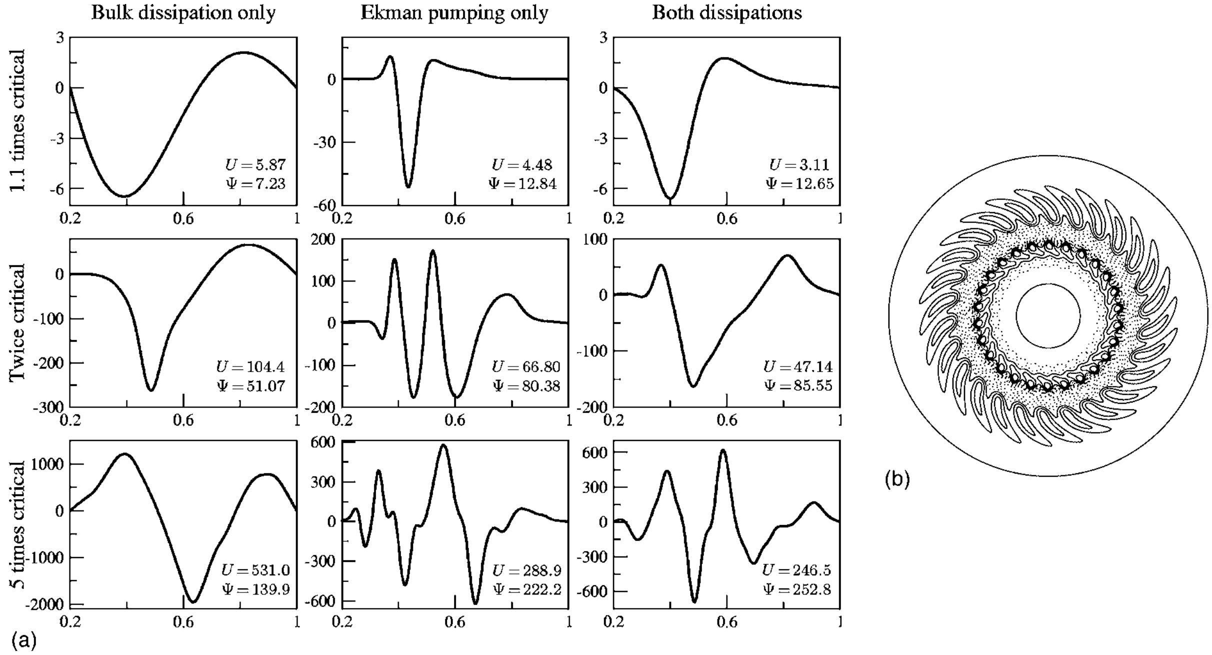

FIG. 1. (a) Zonal wind profiles vs r/r_o for $E=10^{-6}$, and increasing Rayleigh numbers, from top to bottom: $Ra=1.1\times Ra_c$, $Ra=2\times Ra_c$, and $Ra=5\times Ra_c$. The column on the left corresponds to simulations with bulk viscosity only, the middle column to simulations retaining Ekman pumping only, and the right column to simulations combining both effects. The rms value (in space and time) for the zonal (U) and nonaxisymmetric (Ψ) components of the flow is indicated in each graph. (b) Equatorial stream function of the flow for $E=10^{-6}$ and $Ra=2\times Ra_c$ retaining Ekman pumping only.

The $\mathcal{O}(E^{1/3}\mathcal{L})$ radial length scale appearing in the zonal flow u_0 invalidates the large-scale assumption made by neglecting bulk viscosity in (1c). At such small length scales, bulk viscosity becomes dominant. We have therefore performed a third simulation retaining both the *B-V* and the *E-P* terms in (1c). The effect of bulk viscosity in increasing the radial length scale is evident in Fig. 1(a). Bulk viscosity will enlarge the width of zonal bands, as it did in the first case (retaining its effect only). In this case, however, it will only act to enlarge these structures until it becomes comparable to the Ekman pumping. One can easily estimate the transition scale above which Ekman pumping dominates and

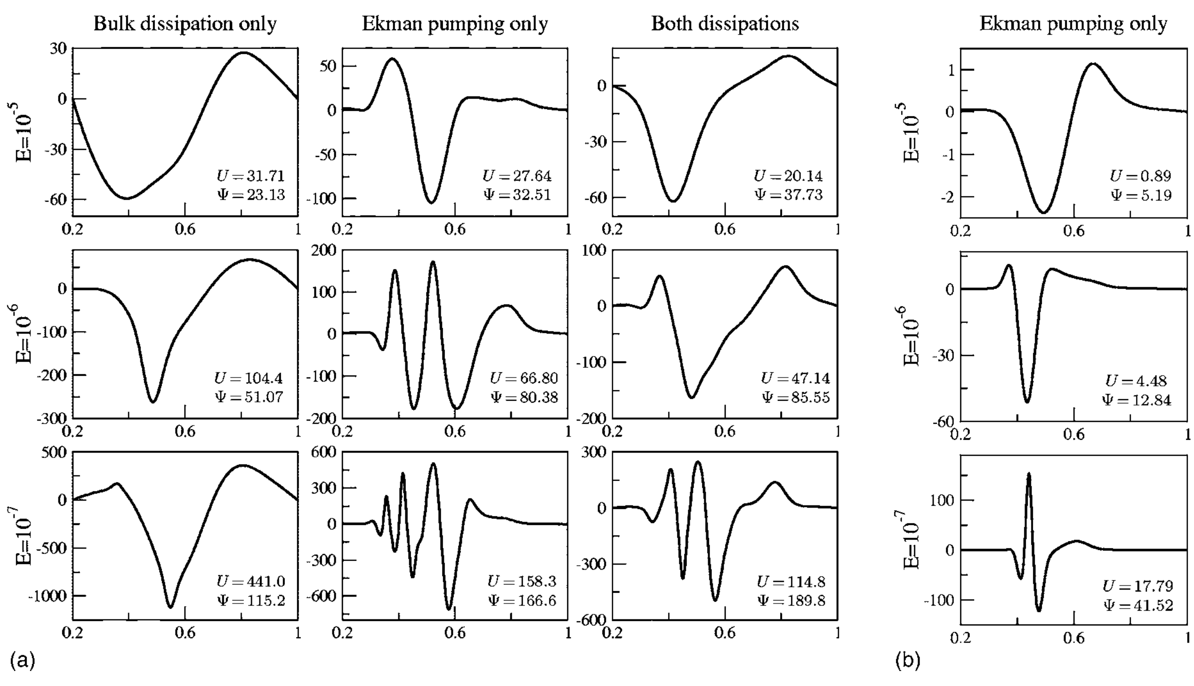

FIG. 2. (a) Zonal wind profiles vs r/r_o at Rayleigh number twice critical ($Ra=2\times Ra_c$), for decreasing Ekman numbers, from top to bottom: $E=10^{-5}$, $E=10^{-6}$, and $E=10^{-7}$ (column arrangement follows that of Fig. 1(a)). (b) Zonal wind profiles vs r/r_o for Ekman numbers: $E=10^{-5}$, $E=10^{-6}$, and $E=10^{-7}$. The Rayleigh number is only 10% above critical ($Ra=1.1\times Ra_c$). These simulations retain the Ekman pumping term (*E-P*) as the only dissipation term on the zonal flow, in order to highlight the increase in the number of bands observed just above the onset with decreasing Ekman numbers.

below which bulk dissipation takes over. Comparing the bulk dissipation at a scale ℓ estimated by $E\ell^{-2}U$ (where U is a measure of the zonal flow velocity) to the Ekman pumping estimated by $E^{1/2}U$ reveals that the transition scale is $\ell \sim \mathcal{O}(\mathcal{L}E^{1/4})$. Any smaller scale will be predominantly affected by bulk viscosity; any larger scale predominantly senses Ekman friction.

To illustrate the Ekman dependence in our simulations, we now present results obtained for a Rayleigh number twice critical, varying the Ekman number from 10^{-5} to 10^{-7} (see Fig. 2(a)). This wide amplitude of variation is only made possible by the model's simplicity. Despite the use of intensive computing, smaller values of E are at present not attainable.

Figure 2(a) clearly demonstrates the faster increase in the number of bands, when the E-P term only is retained, than in the presence of both the E-P and B-V terms. The distinction between the $E^{1/3}$ and the $E^{1/4}$ scaling is however difficult to establish at these values of E. A rigorous distinction between these scalings would require the computation of even smaller values of the Ekman number (these scalings only differ by a factor of $E^{1/12}$).

The increasingly important role given to the zonal flow near the onset as the Ekman number is decreased (as identified in Ref. 2) led us to investigate the variation in the banded structure near the onset with decreasing Ekman number. Indeed, the number of bands increases with decreasing Ekman numbers even very near the onset. This effect is highlighted by Fig. 2(b) at a Rayleigh number 10% above critical. Because of the moderate values of E that can be achieved (although significantly lower than can be achieved with full 3D modeling), we represent here the simulation with the E-P term only (for which bands are narrower). The effect is then more visible near the onset because of the steep $E^{1/3}$ scaling. The number of bands as well as the zonal flow amplitude increase with decreasing Ekman numbers. Direct comparison of Fig. 2(a) with Fig. 2(b) suggests that the Ekman number controls the jet width, whereas the Rayleigh number determines the width of the region influenced by the zonal shear (i.e., the envelope of the banded structure). This property of convergence toward the onset validates the weakly nonlinear approach.

The mechanisms for banded structure formation reported here exist in the weakly nonlinear regime. As such, they contrast with the description of zonal winds associated with turbulent effects (e.g., Rhines[11]). It is to be stressed that J. Rotvig (private communication) also derives an $E^{1/4}$ scaling of the bands, but in the strongly nonlinear regime, deriving this result from Rhines' scaling. Despite the simplicity of our model, the banded structure inevitably suggests an application to the zonal flows observed in giant planets. Deep convection models producing bands in the sphere are limited so far to a moderate number of bands per hemisphere.[3,13–18] Investigation of decaying turbulence in a full sphere[18] has allowed more importance to be given to nonlinearities and also revealed the presence of a banded structure. The possible deep origin of bands in giant planets therefore remains a much debated topic. The model we investigate is very simplified and well justified only near the onset of convection. Besides, it cannot capture the complicated structure of these planets. The only conclusion that can be drawn from our simple model in that respect is that, while turbulence is clearly present in giant planets, it does not appear to be a necessary ingredient to form banded structures. Our simple weakly nonlinear model, retaining the interactions between two modes only, shows that provided Ekman pumping is included, bands can be obtained in rapidly rotating convection (i.e., in the limit of small Ekman numbers) immediately above the onset of convection.

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