

Relations between the dynamo region geometry and the magnetic behavior of stars and planets

L. GOUDARD and E. DORMY^(a)

MAG (IPGP & ENS), CNRS UMR7154, LRA Département de Physique - 24 rue Lhomond, 75005 Paris France, EU

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Abstract – The geo and solar magnetic fields have long been thought to be very different objects both in terms of spatial structure and temporal behavior. The recently discovered field structure of a fully convective star is more reminiscent of planetary magnetic fields than the Sun's magnetic field (DONATI J.-F. *et al.*, *Science*, **311** (2006) 633), despite the fact that the physical and chemical properties of these objects clearly differ. This observation suggests that a simple controlling parameter could be responsible for these different behaviors. We report here the results of three-dimensional simulations which show that varying the aspect ratio of the active dynamo region can yield a sharp transition from Earth-like steady dynamos to Sun-like dynamo waves.

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Introduction. – Observations of the magnetic fields due to dynamo activity appear to fall into two categories: fields dominated by large-scale dipoles (such as the Earth and a fully convective star), and fields with smaller-scale and non-axisymmetric structures (such as the Sun). Moreover two kinds of different temporal behavior have been identified so far: very irregular polarity reversals (as in the Earth), and quasi-periodic reversals (as in the Sun). Since the Earth and the Sun provide the largest database of magnetic-field observations, these objects have been well studied and described in terms of alternative physical mechanisms: the geodynamo involves a steady branch of the dynamo equations, perturbed by strong fluctuations that can trigger polarity reversals, whereas the solar dynamo takes the form of a propagating dynamo wave. The signature of this wave at the Sun's surface yields the well-known butterfly diagram (sunspots preferentially emerge at a latitude that is decreasing with time during the solar cycle).

Modelling. – Because of their very different natures (liquid metal in one case, plasma in the other), planetary and stellar magnetic fields are studied by different communities. Non-dimensional numbers controlling the dynamics of the Earth and the Sun, for example, do significantly differ (see [1,2]). As a practical matter, however, the techniques as well as the typical parameters used in numerical studies of these two systems are surprisingly

similar. To some extent this is due to the restricted parameter space available to present-day computations. The parameter regime numerically accessible is rather remote from the actual objects. For planetary dynamos the main discrepancy relies in the rapid rotation in the momentum equation (characterized by the *Ekman number*), whilst for stellar dynamos it relies in solving the induction equation with weak resistive effects (characterized by high values of the *magnetic Reynolds number*). Yet within this restricted domain, the sharply different key characters to both geo [3] and solar [4,5] magnetic fields have been reproduced. This leads us to argue that the important parameter controlling the magnetic-field behavior is the aspect ratio of the dynamo region (*i.e.* the radius ratio of the inner bounding sphere to the outer bounding sphere). Indeed, in the Earth, the inert solid inner core extends to less than 40% of the core radius, whereas in the Sun, the radiative zone fills 70% of the solar radius. One expects the convective zones of stars and planets to have all possible intermediate aspect ratios, even extending to fully convective spheres.

In order to isolate and understand this purely geometrical effect, we have carried out three-dimensional numerical simulations of self-excited convective dynamos in which the domain aspect ratio was slowly varied, with all other parameters held constant. The governing equations as well as parameter regimes used here were originally introduced for a geodynamo reference calculation [6]. The only distinction being the use of stress-free boundary conditions on the outer sphere of the domain, while imposing

^(a)E-mail: dormy@phys.ens.fr

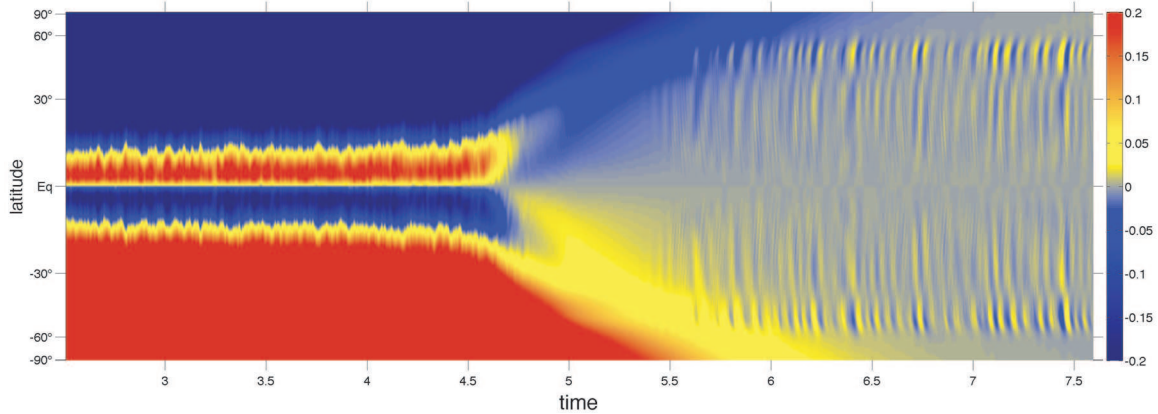


Fig. 1: Time evolution of the radial magnetic field averaged in longitude (for an aspect ratio of 0.65). The initial dipole field survives for a few diffusion times, and then vanishes to yield a butterfly-like diagram.

no-slip boundary conditions at the bottom of the convective region. This choice was made in order to create a strong shear at the base of the model, and thus try to mimic the solar tachocline [7]. The inner sphere is here assumed to be insulating, and we use differential heating. The governing equations are in non-dimensional form:

$$E [\partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla) \mathbf{u}] = -\nabla \pi + E \Delta \mathbf{u} - 2\mathbf{e}_z \times \mathbf{u} + \widetilde{Ra} \mathbf{r} \theta + Pm^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B}, \quad (1)$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B}) + Pm^{-1} \Delta \mathbf{B}, \quad (2)$$

$$\partial_t \theta + (\mathbf{u} \cdot \nabla)(\theta + T_s) = Pr^{-1} \Delta \theta, \quad (3)$$

$$\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{B} = 0, \quad (4)$$

where

$$E = \frac{\nu}{\Omega D^2}, \quad \widetilde{Ra} = \frac{\alpha g \Delta T D}{\nu \Omega}, \quad Pr = \frac{\nu}{\kappa}, \quad Pm = \frac{\nu}{\eta}. \quad (5)$$

All simulations reported here were performed keeping the following parameters constant $E = 10^{-3}$, $\widetilde{Ra} = 100$, $Pr = 1$, $Pm = 5$. The above system is integrated in three dimensions (3D) of space using the Parody code [8].

When the inner (non-dynamo-generating) body occupies less than about 60% of the convective body in radius, the flow generates a dipolar field, very similar to that of the Earth. It features patches of intense flux at high latitudes and some reversed patches at low latitude, similar to the ones revealed by a downward continuation of the Earth's field to the core-mantle boundary [9]. This strongly dipolar solution becomes unstable with a further increase of the aspect ratio. For an aspect ratio of 0.65 —close to that of the Sun— the strong dipole is first maintained and then strongly weakens, but the dynamo action continues in a different form: that of a wavy solution with quasi-periodic reversals (fig. 1), reminiscent of some aspects of the solar magnetic-field behavior. Drifting features can be observed both on the radial field at the surface of the model (figs. 1 and 2b) and on the azimuthal (east-west) field below the surface of the model (fig. 2c). Due to the

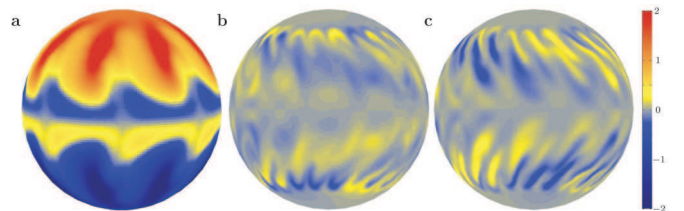


Fig. 2: Radial magnetic field at the surface of the outer sphere, for aspect ratios of 0.45 (a) and 0.65 (b). Azimuthal magnetic field below the surface of the 0.65 aspect ratio model (c).

complex nature of these fully tri-dimensional simulations, many waves can co-exist. Some of the dominant structures appear to propagate toward the equator; others propagate poleward. Reversed waves are also observed at the surface of the Sun at higher latitudes [10]. Let us stress, however, that the model cannot be expected to capture all the features either of the geo or solar magnetic fields. In particular due to the parameters regime and the lack of stratification in our modelling.

Physical interpretation. — In order to investigate the physical mechanisms associated with these waves, we have performed some kinematic simulations. During the course of the simulation the Lorentz force was suppressed. The wavy nature of the dynamo field was unaltered by this modification. This rules out the possibility of an interpretation in terms of pure Alfvén waves or Alfvén waves modified by rotation (so-called MC or MAC waves), which both require the back-reaction of the Lorentz force. Of course, suppressing the Lorentz force is not without consequences: the flow slowly evolves to a different purely hydrodynamical state, and the magnetic field now grows exponentially, but both of these effects are sufficiently slow for the wavelike character to persist over many wave periods.

Two other interpretations for the nature of these waves remain possible: either hydrodynamic fluctuations (*e.g.* inertial waves or Rossby waves) or dynamo waves, as expected on the Sun. These possibilities were tested by

comparing oscillations in the velocity field and in the magnetic field in the kinematic simulations. We found that a high-frequency signal is present both in the flow and in the magnetic field. This demonstrates the presence of hydrodynamic waves, which induce magnetic fluctuations. The lower-frequency signal is however absent in the flow. This provides a proof of their “dynamo wave” nature.

We have numerically observed such dynamo waves for aspect ratio up to 0.8. For the parameters investigated here, the transition from a dynamo dominated by a fluctuating dipole to a dynamo wave occurs for an aspect ratio close to 0.65. This transition exhibits hysteresis: once a dynamo wave solution is present, the aspect ratio can be reduced again down to 0.6, while maintaining this dynamo mode.

Connections with parameterized models. – Butterfly diagrams indicative of the solar cycle are usually produced using simplified parameterized models or “mean-field” models. These models require a prescription of the turbulent induction, the so-called “ α -effect” (which can also be introduced in terms of deviation from axisymmetry [11]). We should stress that this is a valid approximation only if certain conditions are satisfied (*e.g.*, [12]). Such butterfly-like diagrams are generally not produced by direct three-dimensional modelling, with the notable exception (only in the reverse direction) of the pioneering work of Gilman and Glatzmaier [4,13].

Because of the strong symmetry of the convective flows influenced by the rapid rotation of the planet or the star, it is well known that two independent families of solutions exist, namely with dipole symmetry (antisymmetric with respect to the equator) and quadrupole symmetry (symmetric with respect to the equator). Both families of solutions are often described in reduced parameterised models [14,15], and we have observed these two families in our fully 3D simulations (fig. 3). Both branches are stable in our simulations for long periods of time, but can also be destabilised to yield a change of symmetry. In fact, despite the relative complexity of our model, the temporal behavior of both symmetries is clearly reminiscent of kinematic studies of earlier reduced models (fig. 4c, d and [14]).

The simpler mean-field equations for the axisymmetric field are obtained by writing the flow and field as

$$\mathbf{u} = s\omega \mathbf{e}_\phi, \quad \mathbf{B} = \mathbf{B}_p + B \mathbf{e}_\phi = \nabla \times (A \mathbf{e}_\phi) + B \mathbf{e}_\phi, \quad (6)$$

i.e. assuming a mean flow in the form of a zonal shear only. In the isotropic case, the axisymmetric part of (2) yields (*e.g.*, [2])

$$\frac{\partial A}{\partial t} = \alpha B + \text{Rm}^{-1} \mathcal{D}_2 A, \quad (7)$$

$$\frac{\partial B}{\partial t} = s \mathbf{B}_p \cdot \nabla \omega + (\nabla \times \alpha \mathbf{B}_p) \cdot \mathbf{e}_\phi + \text{Rm}^{-1} \mathcal{D}_2 B, \quad (8)$$

where s denotes the cylindrical radius and $\mathcal{D}_2 = \Delta - 1/s^2$ (note that Pm in (2) is here changed to Rm as the flow is now assumed to be given).

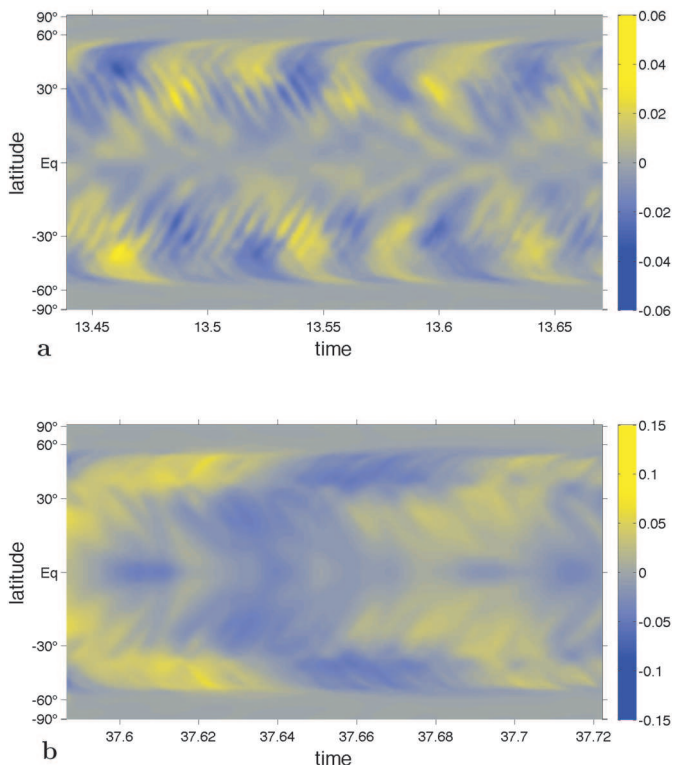


Fig. 3: Time evolution of the zonal average of the azimuthal magnetic field below the surface of the model, for an aspect ratio of 0.65: the antisymmetric (a) and symmetric (b) solutions.

For an instability of eqs. (7), (8) to exist, these equations must not decouple (this is the essence of Cowling’s anti-dynamo theorem [16]). Equation (8) involves A through two terms. Reduced models have been classified in two categories depending on the dominant term. The first term on the RHS of (8) involves the zonal shear and is referred to as the Ω -effect. The second term in the RHS of (8), as well as the first term on the RHS of (7), involve mean induction from non-axisymmetric features in the flow and are referred to as the α -effect.

Dropping the α -effect term in (8) and writing the resulting equations in a simplified Cartesian geometry yields

$$\frac{\partial A}{\partial t} = \alpha B + \text{Rm}^{-1} \Delta A, \quad \frac{\partial B}{\partial t} = G \frac{\partial A}{\partial x} + \text{Rm}^{-1} \Delta B, \quad (9)$$

where $G = du_y/dz$. Parker [17] was the first to identify travelling-waves solutions (dynamo waves) of the above system. These oscillatory dynamos, named Parker waves, were obtained by Roberts [14] for nearly axisymmetric dynamos in spherical geometries (following the formalism of Braginsky [11]). It was found that while the $\alpha\Omega$ -dynamos tended to be oscillatory (complex growth rate), for α^2 -dynamos the simplest dipole solutions tended to be stationary (real growth rate). A similar behavior can easily be traced in the simpler Cartesian example above (see also [18,19] for a discussion of the generic behavior of such nearly axisymmetric mean-field dynamos).

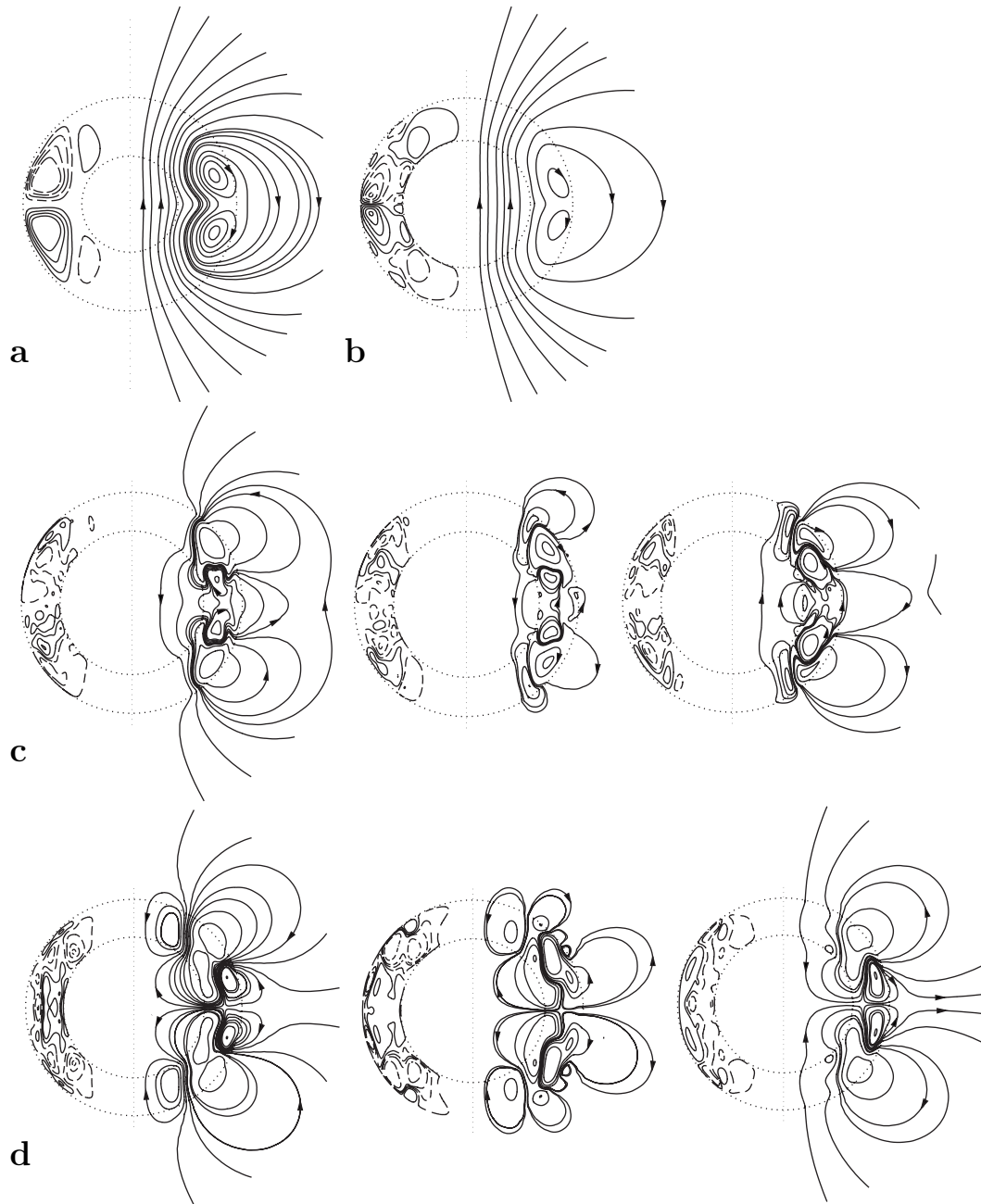


Fig. 4: The zonal average of the magnetic field in our 3D simulations. Contours of the toroidal (east-west) part of the field are plotted in the left hemisphere and the lines of force of the meridional (poloidal) part of the field are plotted in the right hemisphere. The aspect ratio is increased from 0.45 (a) to 0.6 (b) and to 0.65 (c-d). The sequence of dynamo waves is represented for the antisymmetric mode (c) and symmetric mode (d). It is similar in nature to that produced by parameterized models [14].

We can perform further comparisons with reduced models by studying only the axisymmetric component of the simulated field. Figure 4 shows the azimuthally averaged field for some of our fully 3D simulations. The Earth-like mode is represented for aspect ratios of 0.45 and 0.6 (a and b). The active dynamo region lies outside the tangent cylinder [3], it therefore gets increasingly constrained as the inner sphere is increased. The dipole eventually drops for large aspect ratio, when the volume outside the tangent cylinder becomes too small. Weakly

dipolar solutions were also obtained at large aspect ratio in simulations using equations modified by hyperviscosity [20]. The dipolar solution was also found to decay and eventually vanish by increasing the aspect ratio in a reduced parameterized model for the Earth's core [21]. Here we show that the steady dynamo branch can be replaced, at larger aspect ratio, by an oscillatory dynamo mode. A comparison with reduced parameterized models can help interpret this transition to the solar-like mode. A strong zonal wind develops, in our simulations, in the

solar-like mode. Although the terminology of parameterized models must be used with care for direct simulations (the hypothesis of scales separation does not strictly apply), this suggests a transition from a dynamo of the α^2 type to a dynamo of the $\alpha\Omega$ type as the aspect ratio is increased. Indeed, Earth-like three-dimensional models have been interpreted in terms of regeneration by convective vortices only, and thus closer to the α^2 formalism [22] (sometimes referred to as “giant α -effect”), whereas the $\alpha\Omega$ formalism provides the classical framework to model solar dynamo waves, as guided by the strong shear at the base of the convection zone [14,15]. Such nearly axisymmetric dynamos [14] produce cyclic magnetic behavior very similar to the cycles exemplified in fig. 3.

Conclusions. – By varying the aspect ratio, we have observed a sharp transition from a dipole-dominated large-scale magnetic field to a cyclic dynamo with a weaker dipole. This indicates that the geometry of the dynamo region severely constrains the existence of the dipole-dominated solution. We should, however, stress that other parameters, involving the ratio of typical forces, could affect the precise value of the critical aspect ratio for the transition. The values of these parameters in our simulations (as in all numerical models to date) are indeed very remote from the actual relevant values for the Sun, or for the Earth. The potentially strong effect of this parameter change on the dynamo solution should not be underestimated. It is indeed quite striking that, despite these shortcomings, numerical models can capture a good part of the qualitative features of the solar and geomagnetic fields.

Recent observations of stellar magnetism appear to corroborate this mechanism. Donati *et al.* [23] reported observations of a strongly dipolar field in a fully convective star (V374 Peg). More recently, Donati *et al.* [24] have reported magnetic observations of τ -Bootis, a rapidly rotating F star, *i.e.* one with a relatively shallow outer convection zone. Not only did they observe a rather complex magnetic-field structure, but they also report that the overall polarity of the magnetic field has reversed after one year of observation. They interpreted this observation as an indication that the large aspect ratio τ -Bootis star is undergoing magnetic cycles, similar to those of the Sun.

Further observations of planets and stars are needed, but clearly the observations available so far seem to confirm the important role of the aspect ratio in controlling the transition from steady to cyclic dynamo modes.

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REFERENCES

- [1] ZHANG K. and SCHUBERT G., *Rep. Prog. Phys.*, **69** (2006) 1581.
- [2] TOBIAS S. and WEISS N. O., in *Mathematical Aspects of Natural Dynamos*, edited by DORMY E. and SOWARD A. M. (CRC Press) 2007.
- [3] GLATZMAIER G. A. and ROBERTS P. H., *Nature*, **377** (1995) 203.
- [4] GILMAN P. A., *Astrophys. J. Suppl. Ser.*, **53** (1983) 243.
- [5] BROWNING M. K., MIESCH M. S., BRUN A. S. and TOOMRE J., *Astrophys. J. Lett.*, **648** (2006) L157.
- [6] CHRISTENSEN U. *et al.*, *Phys. Earth Planet. Inter.*, **128** (2001) 25.
- [7] HUGHES D. W., ROSNER R. and WEISS N. O. (Editors), *The Solar Tachocline* (Cambridge University Press) 2007.
- [8] DORMY E., PhD Thesis (1997); DORMY E., CARDIN P. and JAULT D., *Earth Planet. Sci. Lett.*, **160** (1998) 15; [6] and later collaborative developments.
- [9] CHRISTENSEN U., OLSON P. and GLATZMAIER G. A., *Geophys. J. Int.*, **138** (1999) 393.
- [10] HATHAWAY D. H., in *Proceedings of SOHO 12 GONG + 2002*, edited by SAWAYA-LACOSTE H., ESA SP-517 (2003) p. 87.
- [11] BRAGINSKY S. I., *Sov. Phys. JETP*, **20** (1964) 726; (1965) 1462.
- [12] HUGHES D. W. and CATTANEO F., *J. Fluid Mech.*, **594** (2008) 445.
- [13] GLATZMAIER G. A., *Astrophys. J.*, **291** (1985) 300.
- [14] ROBERTS P. H., *Philos. Trans. R. Soc. London, Ser. A*, **272** (1972) 663.
- [15] BEER J., TOBIAS S. and WEISS N. O., *Solar Phys.*, **181** (1998) 237.
- [16] COWLING T. G., *Mon. Not. R. Astron. Soc.*, **94** (1934) 39; see also ZELDOVICH YA. B., *Sov. Phys. JETP*, **4** (1957) 460.
- [17] PARKER E. N., *Astrophys. J.*, **122** (1955) 293.
- [18] MOFFATT H. K., *Magnetic Field Generation in Electrically Conducting Fluids* (Cambridge University Press) 1978.
- [19] PARKER E. N., *Cosmical Magnetic Fields* (Clarendon Press, Oxford) 1979.
- [20] GOMEZ-PEREZ N. and HEIMPEL M., *Geophys. Astrophys. Fluid Dyn.*, **101** (2007) 371.
- [21] JAULT D., *C. R. Acad. Sci. Paris Ser. IIA*, **323** (1996) 451.
- [22] OLSON P., CHRISTENSEN U. and GLATZMAIER G. A., *J. Geophys. Res.*, **104** (1999) 10383.
- [23] DONATI J.-F., FORVEILLE T., COLLIER CAMERON A., BARNES J. R., DELFOSSE X., JARDINE M. M. and VALENTI J. A., *Science*, **311** (2006) 633.
- [24] DONATI J. F., MOUTOU C., FARÈS R., BOHLENDER D., CATALA C., DELEUIL M., SHKOLNIK E., CAMERON A. C., JARDINE M. M. and WALKER G. A. H., *Mon. Not. R. Astron. Soc.*, **385** (2008) 1179.