Tuned into Earth

The images obtained through various techniques like MRI or magnetoencephalography also require sophisticated mathematics to be interpreted. Is this white patch on an MRI a benign or a malignant tumor? The image alone cannot give the answer. "The problem is that MRIs give no information on physical parameters like hardness or electrical conductivity that can point to a tumor," says Habib Ammari of the DMA-ENS,[3] who is developing mathematical and numerical models for new medical imaging systems. This is true of a wide range of other imaging techniques. To help doctors make a diagnosis, mathematical models of the response of brain tissue to imaging methods have therefore been developed. These models help anticipate how diseased regions will look like on the actual images.

Another way to improve diagnoses is emerging, namely the combination of different imaging techniques. Together with teams led by Mathias Fink and Claude Boccara, both physicists at the Institut Langevin[4] in Paris, Ammari and his group have developed new imaging methods that combine the use of sound waves (as in ultrasound scans) and electromagnetic waves. When applied to breast cancer, these methods show excellent selectivity, giving virtually no false positives and providing incomparably sharp images. In the future, similar combinations of multi-wave techniques could very well bring about radical changes in medical imaging. Since reconstructing images is a highly complex undertaking, mathematicians will play a key role in this revolution.

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In Earth sciences as in other fields, mathematics is key to helping researchers untangle the complexities of our planet, as shown in the following examples.

The first one concerns tsunamis, a phenomenon that regularly affects the wellbeing—and often survival—of populations of the Pacific and Indian Oceans. How do these deadly waves form, and how do they travel across the ocean? Is it possible to prevent the flooding they cause? In an effort to answer these questions, several tsunami prevention groups throughout the world use hydrodynamic numerical models. But classical models frequently get bogged down in the calculations, and are incapable of dealing with atypical situations, such as extremely jagged coastlines or highly uneven seabeds.

The VOLNA numerical model developed in 2008 by Denys Dutykh and his colleagues at the Mathematics Laboratory[1] of the University of Savoie has no such drawbacks. "Our model uses the latest advances in numerical computing, which had never been used before in this field," Dutykh explains. This model is also able to correctly reproduce tsunamis that propagate across the ocean while the sea floor is still active, like the one that struck the Indonesian island of Sumatra in 2004. It was caused by an earthquake that lasted 10 minutes and led to an extremely complex situation in

13 Diagram of the thermal convection movements that take place within the Earth's core.

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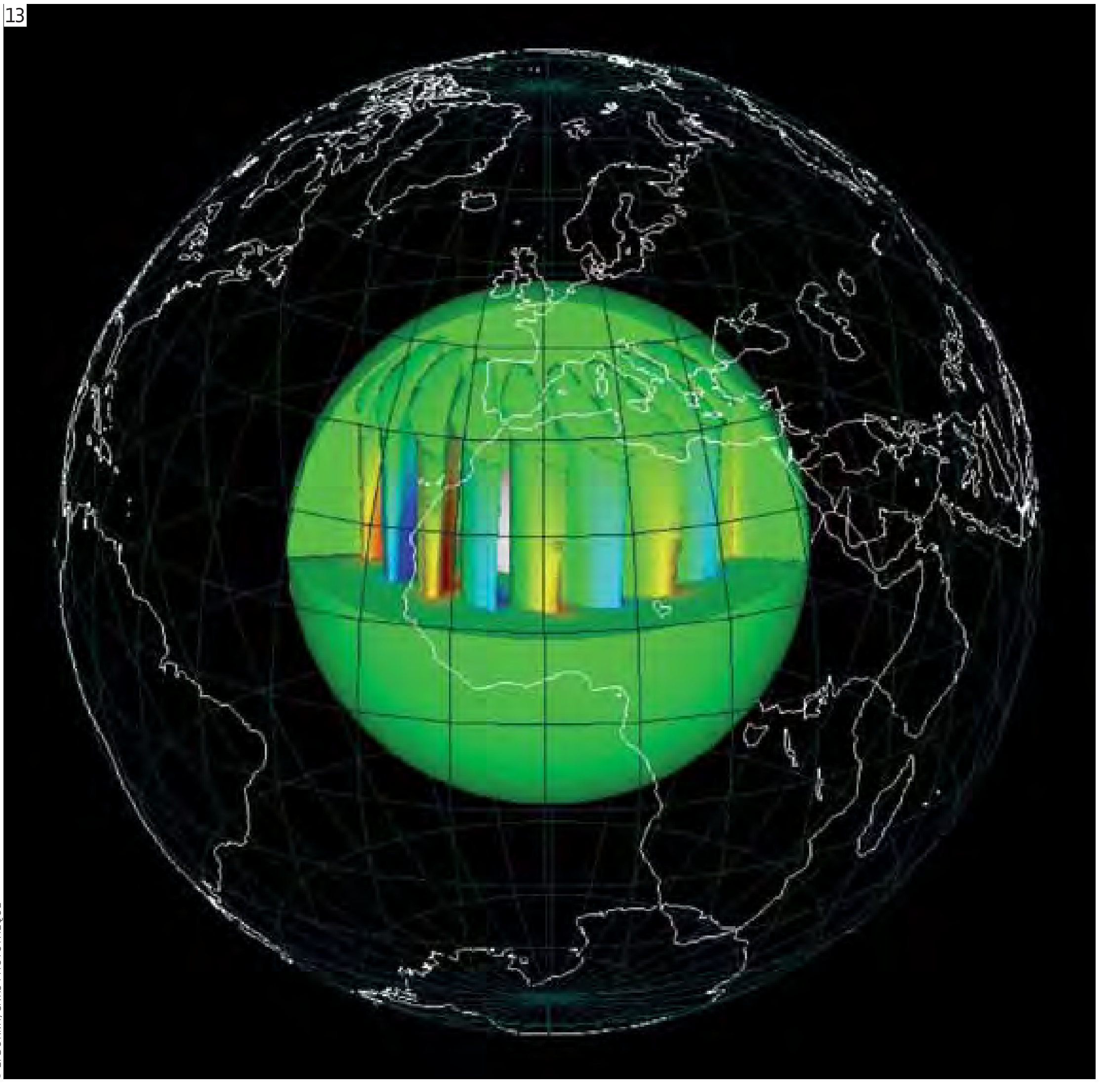

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terms of propagation. But earthquakes are not the only natural disasters that generate tsunamis. The team studied the case of submarine landslides in the Saint Lawrence River in Quebec (Canada). Such landslides give rise to high waves that then sweep onshore, flooding homes. The researchers were able to use their expertise to produce flood maps for the areas at risk.

INVALUABLE ALGORITHMS

Water can also be devastating in landlocked areas. Although the consequences are less dramatic, the runoff of rainwater from fields is a real scourge for farmers and local residents. Runoff can wash up to several tens of tons of earth down-

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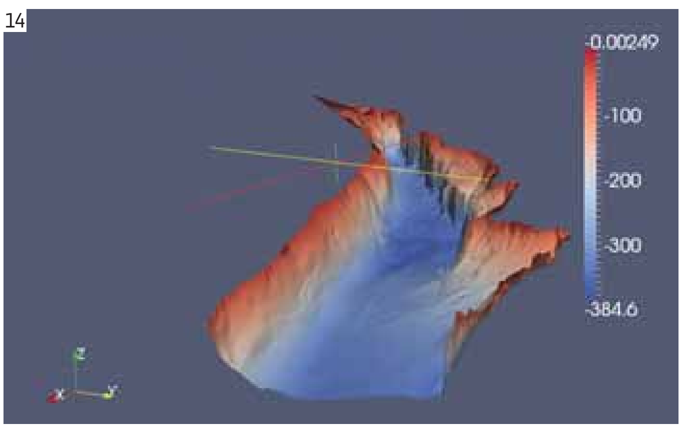

14 Map of the Saint Lawrence riverbed, in Canada. This data was used by Denys Dutykh to study flood risks resulting from river landslides.

stream per year and per hectare, leading to a significant fall in crop yields, or causing flooding and mudslides. A number of anti-erosion measures have been taken in France over the past few years, such as planting vegetation during cropless periods—so that the ground is not left bare—or restoring hedgerows. But are such measures really effective? That's what the METHODE project, which brings together hydrologists, mathematicians, agronomists, and computer scientists, is trying to find out by developing novel numerical models of surface runoff.

"What's difficult about simulating the flow of water across a plot of land is that the sheet of water has a thickness similar to that of the imperfections of the ground, basically a few centimeters," explains Cédric Legout, of LTHE[2] in Grenoble. The presence of small-scale turbulence, the separation of the flow into pools, and the flooding of furrows in the ground make it difficult to predict the total runoff from the plot. It is up to the group's physicists to simplify these phenomena, and identify those that play a leading role on a given scale. "The mathematicians then have to ensure that the algorithms are compatible with the physicists' simplifications," Legout adds. Scientists hope that such research could make planning policies more efficient in the future, thus protecting people and crops from water's devastating effects.

AVOIDING OVERSIMPLIFICATION

It is not always possible to strip a physical system down to its bare essentials. This is true for the convective motions within the Earth's core, which produce the terrestrial magnetic field through a self-excited dynamo effect. The origin of the field and some of its properties—it reverses approximately every 100,000 years—are still debated. "We have a series of mathematical theorems that show that if the models of the terrestrial dynamo are oversimplified, these mysteries become impossible to solve," explains Emmanuel Dormy of IPGP/ENS.[3]

The research group he belongs to studies phenomena such as ocean and atmospheric circulation, or the terrestrial dynamo which involve flows of matter on different scales, but that have a mutual effect on one another (e.g., ocean currents or winds). The researchers are trying to identify which of the processes involved can be simplified without losing their essential features. Since they can't tackle the geodynamo problem in its entirety, the team has focused on the friction between the flow of liquid metal and the core-mantle boundary, roughly 3000 km below the surface. By doing so, they have reached the counterintuitive conclusion that the friction is less intense on this rough surface than if the same surface had been smooth. It's by stacking up this kind of data that the geodynamo will eventually be deciphered. As the famous saying goes, "the devil is in the details." So is mathematics.

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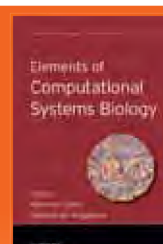

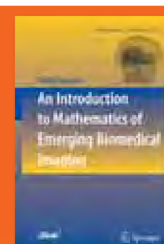

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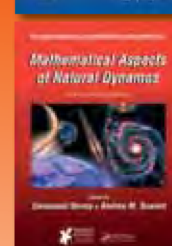

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