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CONTRIBUTORS

Daniel Brito
Lab. Geophysique Interne et Tectonophysique
Batiment IRIGM / U. Joseph Fourier, BP 53
F–38041 Grenoble Cedex 9, France.

Friedrich H. Busse
Institute of Physics
University of Bayreuth
D–95440 Bayreuth, Germany.

Philippe Cardin
Lab. Geophysique Interne et Tectonophysique
Batiment IRIGM / U. Joseph Fourier, BP 53
F–38041 Grenoble Cedex 9, France.

Benoît Desjardins
C.E.A./D.I.F.
B.P. 12
F–91680 Bruyères le Châtel, France.

Emmanuel Dormy
MAG (IPGP–ENS)
Département de Physique, LRA
Ecole Normale Supérieure
24 rue Lhomond
F–75231 Paris Cedex 05, France.

Stephan Fauve
Laboratoire de Physique Statistique
Département de Physique
Ecole Normale Supérieure
24 rue Lhomond
F–75231 Paris Cedex 05, France.

David R. Fearn
Department of Mathematics
University of Glasgow
University Gardens
Glasgow G12 8QW, U.K.

Andrew D. Gilbert
Centre for GAFD
Mathematics Research Institute
University of Exeter
Harrison Building, North Park Road
Exeter, EX4 4QE, U.K.

Raymond Hide
Department of Mathematics
Imperial College London
London SWT 2BZ, U.K.

David W. Hughes
Department of Applied Mathematics
University of Leeds
Leeds LS2 9JT, U.K.

Christopher A. Jones
Department of Applied Mathematics
University of Leeds
Leeds, LS2 9JT, U.K.

Irene Moroz
Mathematical Institute
University of Oxford
24–29 St. Giles'
Oxford OX1 3LB, U.K.

François Pétrélis
Laboratoire de Physique Statistique
Ecole Normale Supérieure
24 rue Lhomond
F–75231 Paris Cedex 05, France.

Michael R.E. Proctor
Department of Applied Mathematics
and Theoretical Physics
University of Cambridge
Wilberforce Road,
Cambridge CB3 0WA, U.K.

Paul H. Roberts
Department of Mathematics
University of California Los Angeles
Los Angeles, CA 90095–1567, U.S.A.

Anvar Shukurov
Department of Mathematics
University of Newcastle
Newcastle, NE1 7RU, U.K.

Radostin D. Simitev
Institute of Physics
University of Bayreuth
D–95440 Bayreuth, Germany.

Andrew M. Soward
Centre for GAFD
Mathematics Research Institute
University of Exeter
Harrison Building, North Park Road
Exeter, EX4 4QE, U.K.

Steven M. Tobias
Department of Applied Mathematics
University of Leeds
Leeds, LS2 9JT, U.K.

Nigel O. Weiss
Department of Applied Mathematics
and Theoretical Physics
University of Cambridge
Wilberforce Road
Cambridge CB3 0WA, U.K.

List of Tables

List of Notations

We use the following notations as a standard. When deviations from this convention is needed, introducing a risk of confusion, this will be clearly specified in the text.

α	turbulent EMF coefficient	34
β	turbulent magnetic diffusivity	36
Δ	Laplace operator or Laplacian	7
η	magnetic diffusivity	7
Θ	temperature perturbation	16
ϑ	density perturbation	165
κ	thermal diffusivity	9
$\widehat{\eta}$	Coriolis parameter	173
Λ	Elsasser number	143
μ	magnetic permeability	4
ν	kinematic viscosity	9
ρ	density	8
σ	electrical conductivity	4
τ	Coriolis number	178
χ	magnetic helicity	91
$\boldsymbol{\Omega}$	rotation vector	9
Ω	angular velocity	11
$\boldsymbol{\omega}$	vorticity	21
$\mathbf{A}$	vector potential	7
$\mathbf{B}$	magnetic induction	4
D	dynamo number	38
$\mathbf{E}$	electric field	4
E	Ekman number	16
E_η	magnetic Ekman number	210
Fr	Froude number	11
$\mathcal{H}$	helicity	64

PREFACE

The idea behind this book originated at a meeting held in Caramulo in Portugal in September 2003. The participants agreed that, though the field of natural dynamos (planetary, stellar and galactic) was rapidly evolving and attracting the interest of researchers in other branches of fluid mechanics, there was no comprehensive introductory book for researchers or graduate students entering this research area. The organisers therefore decided to take advantage of the broad-based knowledge of the invited lecturers at the conference to assemble a “multi-authored monograph”. Despite the obvious contradiction in this phrase, it does reflect the spirit in which this book was prepared. While each section of the book was written by specialists in different aspects of this subject, a concerted effort has been made to provide a unified presentation, which develops concepts in a coherent order and, where feasible, uses consistent notation.

The first part of the book is devoted to the theoretical background that forms the foundation of dynamo theory and which is necessary to describe and understand natural dynamos. The first chapter introduces the governing equations and outlines kinematic dynamo theory. Although linear equations are often considered as simple, the reader will see how even the kinematic theory of dynamo action can raise subtle issues. The second chapter turns to nonlinear effects. These include amplitude saturation, but also intricate dynamics such as polarity reversals. Because of angular momentum conservation, most natural objects are rapidly rotating. This induces very specific effects in their relevant fluid dynamics. These are discussed in the third and last chapter of this part.

In the second part of the book, we turn our attention to natural dynamos and their modelling. Amongst natural dynamos, the one which we know best is without doubt our own planet, the Earth. We therefore begin the fourth chapter with a description of the Earth’s magnetic field and our present understanding of its characteristics. In the following chapter, we turn to the other planets of our solar system. In the sixth chapter we study the magnetic field of stars, including our own star, the Sun. The seventh chapter addresses dynamo action on an even larger scale, that of galaxies.

Finally, we describe experiments conducted over the years which try to model natural dynamos. We conclude with some speculations about future research directions in this rapidly evolving subject.

The understanding of the origin of magnetic fields in astrophysics and geophysics provides a considerable challenge. Each authors in this book conveys their interest and enthusiasm for their individual field of research. We hope that the reader, whatever his background or research experience, will find that this book has reached our desired objective. That is to bridge the gap between mathematicians, physicists, geophysicists and astrophysicists, each working on natural dynamos using their own specific approaches. We hope that the reader will come to enjoy the complexities of this fascinating area of research as much as the many authors of this book do.

Emmanuel Dormy and Andrew Soward.

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ERRATUM

If typos are found in this book, an erratum will be made available at the following website: `http://www.phys.ens.fr/~dormy/MAND`

Part I

Foundations of dynamo theory

CHAPTER 1

INTRODUCTION TO SELF-EXCITED DYNAMO ACTION

Benoît Desjardins, Emmanuel Dormy
Andrew Gilbert & Michael Proctor

The theory of self-excited dynamo action discussed throughout this volume was first suggested by Sir Joseph Larmor in 1919 to account for the magnetic field of sunspots. This concept was later formalised mathematically by Walter Elsasser (1946). The objective of this first chapter is to introduce the subject and provide the necessary background for the later developments. As such, we derive the relevant equations and discuss the usual approximations in Section 1.1, before introducing the concept of a homogeneous self-excited dynamo in Section 1.2. Having dispensed with these preliminaries, the existing theoretical results and necessary conditions for dynamo action are then presented in Section 1.3 and the essential distinction between steady and time-dependent velocities then follows in Section 1.4. In Section 1.5, we then introduce the concept of mean field electromagnetism, which will be a reoccuring topic throughout the book. Finally, in Section 1.6, we discuss the difficult large magnetic Reynolds number limit, which is relevant for astrophysical problems.